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# DEVELOPMENT OF MATHEMATICAL CONCEPTS IN JUNIOR AND SENIOR SECONDARY SCHOOL STUDENTS

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# DEVELOPMENT OF MATHEMATICAL CONCEPTS IN JUNIOR AND SENIOR SECONDARY SCHOOL STUDENTS

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***Abstract***
*The present world requires people with vast skills and abilities who can collect, organize, synthesise, analyse information, and apply the knowledge in solving everyday problem in the society. In essence, students are to be trained and equipped with the appropriate math skills that will enhance mathematical concepts and processes understanding for adaptation to the endless changing technologies, social, economic, and global conditions. This article considered the development of secondary school conceptual understanding via mathematical processes. Seven mathematical processes that will develop the required mathematical skills and facilitate deeper understanding of math are considered. Therefore, it is recommended that teachers should be encouraged to emphasise these processes during math instruction.*

## Introduction

Mathematics is key to many sciences and technology-oriented careers as well as other disciplines such as finance, journalism, sports, etc. This knowledge is needed by all individuals to contribute effectively to the development and prosperity of their society. Thus, a necessity for students to become informed, reasonable, and effective citizens calls for a strong background in mathematics

(NCTM, n.d.). Secondary school students with a strong background in mathematics are prepared for success irrespective of their future ambition. To achieve this success, students need to learn beyond performing procedures or recalling facts to developing a useful and usable knowledge of mathematics such as analyzing, interpreting and critically thinking about math (NCTM, n.d.). Mathematics is a compulsory subject and a requirement for transition from primary to secondary school. Kilpatrick, et al., (2015) emphasized and differentiated between Mathematical Understanding for Secondary Teaching (MUST) and Understanding for Elementary Mathematics Teaching (UEMT) in the following ways:

i. There is a wider range of mathematical content (i.e. more topics are studied);
ii. There is a greater emphasis on formality, axiomatic systems and rigor in regard to mathematical proof;
iii. There is more explicit attention to mathematical structure and abstraction, and
iv. The cognitive development of secondary school students is different from primary school pupils based on the way they reason about concepts and relationships in mathematical systems as well as about conceptually challenging mathematical ideas such as probability and mathematical induction.

Hence, secondary school students needed to be taught conceptual thinking that informs the specific techniques of mathematics to be able to blend into this twenty-first century era.

Mathematics as a subject is taught and emphasized at secondary school to enable students to learn and understand math concepts applicable to daily lives. These concepts are contained in the mathematics curriculum prescribed for Upper Basic Education (JSS) and senior secondary classes in Nigeria, which are to be learnt and mastered by them.

According to Kilpatrick (2015), secondary school mathematics goes beyond facts, routines, and strategies. It consists of a collection of mathematical concepts that are interrelated, ways of representing and communicating these concepts, and various tools for finding the solution to different mathematical problems. Important requirements to understand these concepts include reasoning and creativity which will provide the mathematical competence that will lay the foundation for advanced studies in mathematics and other disciplines. He maintained further that learning of secondary school mathematics can be facilitated by a teacher’s deep understanding of the subject which he referred to as the mathematical expertise and skill possessed by a teacher and utilized for the purpose of enhancing students' understanding, expertise and appreciation for the subject. In view of this, teachers need to develop their skills and abilities for teaching the subject to be able to interpret students' math, develop multiple representations of mathematical concepts, be acquainted with students' previous mathematical understanding, and so on. This will inform the progression in mathematics instruction (Kilpatrick, et al., 2015).

A teacher's proficiency in the knowledge and utilization of mathematics curriculum requires them to identify prerequisite concepts that will enhance learning a new concept and serve as a basis for future learning. As viewed by Kilpatrick, et al., (2015), learning mathematical concepts do not follow a fixed or linear order but rather approached and revisited in many ways. The reason being that mathematical concepts progress in the learner's mind from simple to complex and become more sophisticated with each repetition. Also, the curriculum is composed of not only the mathematical contents but mathematical processes which relate, connect, and operate on these contents, thus, there is the need to also develop these mathematical processes in students to enhance the effective learning of mathematical concepts. This will help breed a new set of learners whom Devlin (2012) referred to as mathematical thinkers who will have a better conceptual understanding of mathematics, its power, scope, appropriate application, and its limitation.

Conceptual understanding according to Kilpatrick, et al., (2015) has been described as "knowing why" of mathematics. This can be demonstrated by deriving formulas without simply retrieving or quoting them from memory, evaluating an answer for reasonableness and correctness and understanding connections in mathematics or formulating a proof. For example, knowing and understanding the source of quadratic formula and its derivation, seeing the connections between right triangle trigonometry and the graphs of trigonometry functions, and so on. It has been shown by much research that students who understand the 'why' of math and not just the 'how' manifest an enhanced, deep and durable understanding more than their mates who only memorize basic procedures of solving mathematical problems (NCTM, n.d.). They can easily recall learnt materials and can also apply them effectively to daily activities and future endeavours. Thus, students' competencies in mathematical concepts are developed to enhance their effectiveness in it and in other disciplines particularly for those terminating their education after their Upper Basic education (JSS 3) and for those continuing into tertiary institutions after their secondary school education (Eshun & Amihere, 2014).

Cognitive Development of Secondary School Students

The junior secondary school students (age 11-13) and the senior secondary school students (age 14-16) belong to the formal operational stage of Piaget's Cognitive developmental theory. The formal operational stage is the fourth stage of Piaget's theory of cognitive development. This stage is characterized by logical, rational, and abstract thinking (Simatwa, 2008). The students in this stage can utilize higher level of thinking to tackle problems, that is, they think logically and abstractly when confronted with problems because math at this stage are symbolically represented. Thinking logically implies they could utilize skills and knowledge learnt and developed to determine an outcome. They can link more abstract and complex ideas with prior knowledge and can also apply such to real life situations.

**What is Mathematics?**

Mathematics is an essential area of learning that provides logical, reliable, and growing body of concepts, makes use of specific language and skills to model, analyse and interpret the world (Ministry of Education (MoE), 2012 as cited in Eshun & Amihere, 2014). The acquisition of these skills will help in assisting students' future careers and benefit society at large. Charles-Ogan and Otikor (2016) viewed math as the bedrock of all scientific and technological investigations as it provides the route to the modern world of science and technology. Mathematics is the science that deals with the logic of shape, quantity and arrangement and it is all around us in everything we do (Byline Manual, 2013). The subject is interconnected such that there is a smooth movement between different representations and mathematical ideas (Gardener, 2014). According to Salman (2005), mathematics is a precursor of scientific discoveries and inventions because it is the basis for any meaningful scientific endeavour. This is because any nation's science and technological development depends on the strong mathematical foundation of her youth.

In Nigeria, mathematics is a compulsory subject required to be passed at both the expiration of Basic Education and Senior Secondary Education. Mathematics in the past focused on performing a calculation or computing an answer but there is now a shift in emphasis from doing to understanding. It is now all about both calculation and understanding of abstract concepts and relationships. Though, calculations and understanding concepts are both important in mathematics learning, it is expected of students to have conceptual understanding, that is, not just learning calculations or procedures to solve problems but mastering the underlying concepts and being able to justify the methods used in solving such problems. As mathematics teaching refers to the interaction between the teacher and the learners that leads to the acquisition of desirable mathematical knowledge, ideas and skills required for daily life (Tali, Mbwas & Abe, 2012), it is important to emphasise its proper understanding in students for effective functioning in all their endeavors.

**What are Mathematical Concepts?**

Mathematical concepts (math concepts) are ideas or mental impressions which are primarily related to computing, quantitative relationship, systematic reasoning, or computations (Charles-Ogan & Otikor, 2016). Mathematical concepts are abstract, complex, and hierarchical in nature. And it increases in abstractness, complexity, and hierarchy as class of the students increases (Cetin, Dane, & Bekdemir, 2012 as cited in Arif, Omer, Fatih, & Meryem, 2016). Concepts are said to be formed when ideas or thoughts are developed based on common properties of objects or events by abstraction (Hanson, Twumasi, Aryeetey, Sam & Adukpo, 2016). Math concepts and skills are therefore developed by students through knowledge and experience during math instruction (AbdRahman, 2006 as cited in Effandi, Mohamad, Siti, & Mazlini, 2010). Its learning follows a developmental progression. This entails understanding the levels of thinking at which students are operating, along with the next level of thinking to be learnt, then providing the right tasks that will suit each student's level to enhance effective learning (Clement & Sarama, 2015). Thus, students' previous knowledge of basic concepts provides a basis upon which the learning of subsequent concepts is based.

A way of deepening a concept taught is by applying it to different contexts of problem solving or cross-curricular links according to Habbard (2018). He maintained that this should be relevant and accessible to students to enhance their learning. He further summarized some major points which teachers need to consider in designing a deep and durable math curriculum as follows: knowing important concepts, making connections, determining previous knowledge of students, slowing down, and using retrieval strategies in problem solving and cross-curricular activities. Students learn to solve increasingly difficult problems. Thus, during math instruction, a teacher should first describe the mathematical concepts, structures and skills considering the developmental progression of the students. This will assist in determining the present level of their thinking and the next level, thus informing the provision of task that will link each level of developmental progression (Clement & Sarama, 2015). Exposing students to opportunities of practicing and mastering the concrete, pictorial and abstract phases of development for each math concept is important for their successful learning. It is therefore important to ensure the mathematical concepts and processes development and understanding at a level commensurate to their development and ability which is the reason of this article.

**Learning and Understanding Mathematical Concepts**

Mathematics is a subject recognized by both the government and society as an essential tool for national development (Awofala, 2017; Jonah, Caleb & Stephen, 2012). It is a big concept that has many smaller mathematical concepts which translate to proper life skills. Thus, math concepts are categories of mathematical knowledge (Fritz, Ehlert & Balzer, 2013, Jonah, Caleb & Stephen, 2012). Mathematical concepts are the 'big idea' of math required to be developed and explored with proper understanding by learners for the achievement of curriculum's objectives such as science and technology promotion, provision of skilled and trained manpower in science and technology and commerce, and the acquisition of the innovative skills and competence that will enhance and lead to the development of the society (Charles-Ogan & Otikor, 2016).

According to CPDD (2007) as cited in Charles-Ogan and Otikor (2016), mathematics concepts include numerical concepts, algebraic, geometrical, statistical, probabilistic, and analytical concepts. The Nigerian senior secondary school mathematics curriculum was also structured in themes and consisted of four major concepts to be learnt by students for effective learning and proper understanding of the subject. They include number and numeration, algebraic processes, geometry, statistics and probability, and calculus (Awofala, Lawal, Arigbabu & Fatade, 2022; NERDC, 2014). Concepts are learnt through developmental progression which describes the path students follow in developing understanding and skill about math topics (Clement &Sarama, 2010) based on their level of thinking. As students naturally follow developmental paths in learning math, efforts should be made by teachers to understand this progression and plan individualized activities that will result in an efficient math learning environment. Understanding of math concepts as maintained by Obanya (2004) cited in Charles-Ogan and Otikor (2016) implies having the required knowledge to show versatility and flexibility in learning and not just the ability to store and produce facts and figures. Thus, concepts are to be developed in students for proper understanding of ideas, meaning and diversified knowledge of the concepts.

These concepts are understood when connected via a progressive and cohesive network. That is, the concepts are to be taught in a progressive manner from simple to complex, hence the thematic structuring of the curriculum's contents (Charles-Ogan & Otikor, 2016). The themes are interrelated such that students' understanding in one theme is dependent and supportive of ideas and concepts in other themes. The themes are divided into sub-themes and various topics with students' learning described using content objectives which when achieved through their mathematical work will facilitate students' concepts understanding. Therefore, It is essential that teachers help students develop and master mathematical concepts, knowledge and skills through conceptual understanding.

**What is Conceptual Understanding in Mathematics?**

Mathematics knowledge involves concepts and procedures understanding together with the process of executing specific actions and operations. Two basic kinds of knowledge in mathematics learning process are conceptual knowledge and procedural knowledge. Conceptual knowledge according to Audrey, Tara and Hye (2013), implies the ability of a student to interconnect mathematics across disciplines, think critically about the contents, and communicate key components in math. In other words, it is the knowledge of meaning of rules, generalisations, relations, and procedures (Awofala, 2017; Arif, Omar, Fatih & Meryem, 2016). Procedural knowledge implies the knowledge of symbols, arithmetical operations, and routine rules, utilized in solving problems (Awofala, 2017; Hiebert & Lefevre, 1986 as cited in Arif, Omar, Fatih & Meryem, 2016). That is, the knowledge emphasising recitation of algorithms and facts. Simply having procedural knowledge, i.e. learning how to perform procedures or recall facts, is not enough; one should be able to analyse, interpret and think critically about the mathematics that is learnt. Hence, conceptual understanding is needed to correctly apply both procedural and conceptual knowledge in mathematics.

Conceptual understanding in mathematics can simply be defined as understanding the mathematics being done. It refers to the understanding of mathematical concepts, operations, and relations. According to Johnson (2005), it refers to an individual's representation of the major concepts in mathematics. It involves the understanding of a network of knowledge which can be learned by deep and reflexive thinking and not by rote. To promote effective math learning, students must be made to think about and use mathematics knowledge in a meaningful way. Conceptual understanding entails identification of concepts and procedures' connections and applications in various contexts. This is because, lack of proper understanding may pose difficulty in determining procedures to use in an unanticipated situation or new content, or complicated situations. It is essential to view math as sensible, useful and doable. Molina (2014) opined that conceptual understanding of students should first be developed before focusing on algorithms and shortcuts. According to him, reliance on algorithms, procedures and shortcuts in just getting the correct answer poses a difficulty for students' deep understanding of complex topics in higher grades. Thus, students' conceptual understanding should be built first because it will develop in them the ability to show detailed steps in a process, explain reasons for the occurrence of such steps and connect the process to related concepts. Once conceptual understanding is got, next is the exposure to more efficient ways of performing these same processes. The essence is to enable them to understand their learning, recall and utilize what they have learnt appropriately in their future endeavours (NCTM, n.d.). A practical example of the shortcoming of shortcuts which does not emphasise conceptual understanding as given by Molina (2014) was the process of simplifying a fraction. The example is given below:

Method 1: $\frac{6}{8} = \frac{3\times2}{4\times2} = \frac{3}{4}$

Method 2: $\frac{6}{8} = \frac{3\times2}{4\times2} = \frac{3}{4}\times\frac{2}{2} = \frac{3}{4}\times 1 = \frac{3}{4}$

Molina's argument was that the method 1 is though shorter but does not emphasise conceptual understanding because it didn't give room for students to understand what really happened. On the contrary, method 2 was termed complete, emphasizing conceptual understanding because of the detailed process manifesting all the appropriate steps. He stated further that the method 2 promotes a conceptual understanding of the multiplicative identity, which is "the basic idea that any number multiplied by 1 remains the same". He indicated that the multiplicative identity is one of the basic algebraic tool utilized in solving equations. He stressed further that the method one will only promote shortcut learning while the method two will facilitate conceptual understanding of students because it ensures they are guided through the complete detailed process for a deep understanding of simplifying a fraction and reasons it works.

Teachers should therefore encourage and engage them in developing a positive disposition towards the subject and empower them mathematically by emphasizing mathematical process skills during math instruction which they can apply to learn math concepts and topics for each class.

**Ways of developing mathematical conceptual understanding in secondary school students**

Mathematics is a subject full of abstract concepts. Introducing students to these abstract concepts without first building a basis for understanding the concepts results in memorization and rote learning which may not have solid basis for further learning. Swan (2005) in his article titled conceptual development lessons, classified formative assessment lessons into two which are concept development and problem-solving lessons. He explained that concept development lessons are geared towards assessing and developing students' understanding of basic concepts via activities that will engage students in classifying and defining, representing concepts in multiple ways, testing, and challenging common misconceptions, and exploring structure. On the other hand, he viewed problem solving lessons as lessons that assess and develop students' capacity in selecting and applying mathematical knowledge in non-routine contexts and allow students to compare and critique alternative approaches to solving a problem.

Understanding a concept involves four mental processes as outlined by Sierpinska (1994) as cited in Swan (2005). It involves: bringing it to the foreground of attention, naming and describing its properties (identifying); identifying similarities and differences between this concept and others (discriminating); identifying general properties of the concept in particular cases of it (generalizing); and perceiving a unifying principle (synthesizing, defining).Audrey, Tara and Hye (2013) opined that most modern mathematics careers now require conceptual skills such as: critical thinking, modeling and application of the content which have strongly imparted mathematics curriculum and assessment. They suggested and stressed a mathematical assessment focusing on the growth of conceptual understanding as a requirement for enhancing the effective learning of present mathematics that encourages higher-order thinking and problem solving. Thus, Clement and Sarama (2015) opined that mathematical experiences should involve mathematical processes or practices such as problem solving, reasoning, communicating, modeling, and connecting which will help students mathematize their world. Mathematical conceptual understanding of students can thus be developed and facilitated via mathematical processes. Mathematical processes are the processes through which students acquire and use mathematical knowledge (Lambdin & Lester, 2010). They are the major processes students need to engage in as they learn math concepts and procedures thereby utilizing the knowledge throughout their secondary school classes. Interaction with the subject through mathematical processes will provide them with the opportunity of connecting facts and procedures to conceptual understanding (Molina, 2014). Their development and utilization in doing mathematics will also deepen students' knowledge and understanding of the subject.

It therefore becomes important to build students' understanding of concepts through the following ways:

i. Reasoning
ii. Reflecting
iii. Selecting tools and computational strategies
iv. Connecting
v. Problem solving
vi. Representing
vii. Communicating

I. Reasoning

This literally implies pondering mentally and drawing a logical conclusion on the basis of assumptions and definitions and seeing not just how something works but also why. It also involves developing and understanding of a situation, context or concept by connecting it with existing knowledge to reveal a 'bigger picture' (NCTM, n.d.). This will assist students in thinking about and utilizing mathematics in meaningful ways. Learning math concepts and solving problems in an organized, analytical, well-reasoned way requires a focus on reasoning. This is because it will enable students to develop ability of explaining the reasoning behind a solution or choice of strategy used. Teachers should therefore focus on developing students' conceptual understanding rather than rote memorization. This involves building the students' ability in thinking deeply about mathematics and reasoning intelligently mathematically. That is, they should be able to pose and answer questions intelligently, distinguish between various kinds of statements (such as definitions, conjectures, examples, etc.), understand and utilize math concepts appropriately.

Reasoning is a way through which students make sense of mathematics. In learning math concepts, there is the need to emphasize reasoning, as the skill assists students in making mathematical conjectures, assess conjectures, justify conclusions, plan and construct mathematical arguments (TIPS4RM, 2005). Students are to be assisted in developing and applying reasoning skills such as recognizing of relationship, generalizing through inductive reasoning, using of counter-examples to make mathematical conjectures and justify conclusions, as well as planning and constructing organized mathematical arguments. For instance, students can make conjectures by: making reasoned guess as to the answer or likely strategy to utilize to solve a problem and give reason(s) for a failed attempted process or solution; using evidence gathered to refine hypothesis and so on. Further, making of inferences, conclusions and justifications by using models and logic to infer/ conclude should be encouraged among students. They should also be able to justify why an answer or approach to a problem is reasonable; make and test generalisations based on investigation or observation; analyse statements and provide examples which support or refute them, and evaluate the mathematical thinking and strategies of others using supporting data to explain reasons why a chosen method and solution are mathematically correct, as well as present arguments in a logical and organized manner.

As pointed out by NCTM (n.d.), students are expected to learn mathematical concepts and justify the reasons for doing so. Thus, students should therefore be engaged and encouraged to ask questions and understand math concepts by not just exploring how but also the why of math. This can be done by allowing them assess mathematical ideas without being told what to do or how to do it, and also encouraging them to solve more exercises, problems or assignments with little or no assistance. They should be made to explore concepts without having been shown what to do. They should be encouraged to develop their own solution methods and share their ideas and explanations. A secondary school teacher can encourage reasoning by asking the question ‘How or why does your solution make sense to you?. This tends to bring into limelight students’ level of understanding about a concept, their belief about it and their response (Crystal, 2016).

Instructional strategies to be employed that will enable students make sense of mathematical concepts involves: asking questions that will require students to hypothesise and make conjectures, e.g. what if…?. Teachers can also facilitate the sharing of reasoning behind students' suggestions thereby helping them to take or refute hypotheses; modelling how to adjust a wrong hypothesis; fostering behaviours during whole-class discussions such as active listening to the reasoning of others and legitimizing errors in the learning process; providing avenues for students to work in small, mixed-ability groups where students experiencing difficulty can view the reasoning and explanation of others; leading students to make generalisations and identify patterns after repeated trials; providing students with one or more numerical examples and putting it side-by-side with the generalization, e.g. the development of a quadratic formula; and asking students to explain reasoning behind each step of a mathematical argument.

II. Reflecting

This literally implies thinking deeply or carefully about something said or done by a person or another. It is an important part of good problem solving as it enables students become good problem solvers when they regularly and consciously reflect on and monitor their thought processes. Students clarify their understanding when completing an investigation or solving a problem by reflecting on and monitoring their own thinking. They can reflect on new skills, concepts and question to discover their connection with previous knowledge by examining questions and demonstrating flexibility in choice of strategy based on the question's nature, monitoring their own progress when solving problem and revising as necessary. This is done via reflection on their own thinking and the thinking of others, evaluation of the reasonableness of an outcome/answer by verifying solutions using different methods; assessment of the effectiveness of strategies and process used or proposing other possible strategies that can be used; or by sharing aspects of a challenging problem. Reflecting on choices and outcomes will enable them improve their methods/approaches and discover new possibilities. Thus, teachers should encourage students to think about/ consider their thinking and point out cognitive dissonance in students who misunderstood a concept so as to change and enhance a better understanding. This they can do by reviewing and summarizing concepts misunderstood and encouraging students to ask themselves 'what if' questions (The seven math processes, n.d).

III. Selecting tools and computational strategies

Tools used in math class help in reducing the time spent on routine tasks and allow more time for concept development. They include measuring tools such as rulers and protractors; physical manipulatives such as fraction strips; and digital learning tools such as graphing tools (The seven math processes, n.d.).

Students should develop the ability to select the appropriate electronic tools, manipulatives, and computational strategies for the performance of particular mathematical tasks, investigation of mathematical ideas and solving of problems (TIPS4RM, 2005, p.6). They are required to understand and know when mental arithmetic, pen-and-paper calculation will be appropriate than technology, thus enabling them select and utilise the appropriate tool for solving problems. Further, they are also to use technology (such as tablets, computers, interactive whiteboard, and calculators) for some tasks when necessary but not for all tasks based on the goal of the lesson. Hence, the need for careful choice, use of physical and digital learning tools including manipulatives and/or technology and computational strategies for exploring, representing and solving problems, to develop and enhance their understanding of new concepts.

Teachers contribute in building students conceptual understanding by introducing new technology/manipulatives and model the use of tools (e.g. large model, overhead or data projector) in ways that will enable them explore and build confidence in learning math. This they do by making available a wide range of tools for students' utilisation during instruction and assessment and also allowing them to practically use these tools to demonstrate and communicate their understanding of a skill or concept. This will enable students develop the necessary skills, strategies and competence to tackle problems and use underpinning thinking skills effectively (Ajao, 2018; TIPS4RM, 2005).

IV. Connecting

This involves linking a new concept or mathematical idea to a previous one. Learning is enhanced when students can connect new mathematical concepts with their previous knowledge (what they already know) and also apply mathematical ideas to real-life situations and other subject or contexts requiring such. This can be done by making connections among mathematical concepts and procedures and relating mathematical ideas to situations or phenomena drawn from other contexts such as; daily life activities, current events/happenings, etc, thus enabling students to see the relevance, usefulness and applicability of math to their everyday life beyond the classroom. This in essence facilitates and deepens mathematical understanding. For example, sample questions for developing connection include the following: This concept reminds me of -----; This problem is similar to -----; How does knowing the formula for the volume of cylinder ($\pi r^2 h$) help in finding the volume of a cone ($1/3\ \pi r^2 h$)?; How does your representation (e.g. diagram, manipulative) connect to-------(e.g. the algebraic solution)?; When could this mathematical concept or procedure be used in everyday life?.

Students are to learn how to apply a strategy or concepts from previous learning in another context within or outside mathematics; connect between new and previous knowledge in order to make sense of math being learnt; and also make connections between/among various representations such as numerical, graphical and algebraic.

Teachers are to assist students in making connections among concepts during math instruction. As stated by Molina (2014), a way of improving math teaching and maximizing instructional time is by finding and using connections among math concepts and ideas. He believed that a new concept has a prior knowledge because knowledge about one topic is usually embedded in another. Thus, recognizing and making connections forms a basis of effective understanding of concepts and processes. They can develop connection making by first activating previous knowledge when introducing a new concept to facilitate a connection between a previous learning and new concepts; giving room for students to explore their own procedures and algorithms, thus monitoring them for correctness; using visuals to connect procedures and concepts, e.g. using a line graph to find the x-intercept; making connections explicitly within math, e.g. notice the similarity between solving simultaneous linear equations by substitution or elimination method (which is making the coefficients of one of the unknown variables to be equal before proceeding to add or subtract the two equations to determine the second unknown variable); and also explicitly between math and everyday students experience, e.g. my brother used length measurement and geometry understanding to produce a piggy box.

For instance, Molina (2014) explained that the conceptual definition of average depends on the deep understanding of the related concepts of multiplication and division. Multiplication is widely defined as repeated addition but this gives a shallow understanding. Ideally, the concept of multiplication is a repeated addition of equal-sized groups. E.g. the expression 2 x 3 implies expanding it to 3 + 3, which represents 2 equal-sized groups of 3 each which is 6. Thus, for a proper understanding of average, there is need to connect it to the division concept. Multiplication and division comprised of three key components which are: a total ($\sum x$), a certain number of groups (n), and a constant size for each group (x). Division is thus about equal sharing once the total is known but in average there is an unknown total. The major difference therefore between division and average is the starting point. In division, there is a known total but in average there is an unknown total. Finding average entails, combining the different-sized groups into one total which is the starting point of a division context. Thus, this connection enables students to understand and realize that division is about equal distribution, then average is about equal redistribution (Molina, 2014). This enhances a conceptual understanding of both multiplication and division as concepts connected to average and not just learning the procedures of solving alone.

Mathematically:

$$\text{Average} = \frac{\sum_{i=1}^{n} x_i}{n}$$

This implies, the summation of $x_i$, i=1 to n; divided by n which is the number of different sized groups.

Hence, a teacher should encourage, appreciate and make significant mathematical connections to related concepts during math instruction to enhance better conceptual understanding and deeper understanding of a process.

V. Problem Solving

Problem solving is simply knowing how to solve problems appropriately. It is an essential process which helps students in achieving their expectations of mathematics. As stated in TIPS4RM (2005), it is key to learning math as it represents the foundation of effective math programmes. Problem solving develops students' understanding of concepts and enables them to apply their math skills in daily activities (Akinsola & Awofala, 2008, 2009; Awofala, 2011, 2016, 2017; Awofala, Fatade & Olaoluwa, 2013; Awofala, Balogun & Olagunju, 2011). There is therefore the need for students to be properly built or led to tackle math problems that are appropriately challenging and also explore various approaches to solving such problems. Students are to be developed through the following problem solving processes which involves:

- Using information to identify and define the question(s) within a problem;
- Making a plan and deciding what information and steps are needed to solve the problems;

- Choosing the appropriate operation(s) for a given problem situation;
- Selecting and applying appropriate strategies ranging from visual, numerical, symbolic, etc.
- Organizing, interpreting and using relevant information;
- Selecting and using appropriate tools and technology;
- Identifying alternative ways of finding solution (Mathematics/instructional strategies, n.d. as cited in Ajao, 2018).

Teachers should provide resources and time for students to gather data, choose strategy, detect patterns and also guide them to apply the chosen strategy appropriately. They are also to model alternative procedures and strategies and facilitate the purposeful sharing of different problem solving strategies for the same problem when appropriate. Students should also be encouraged to tackle tasks that demand sustained effort overtime, e.g. problem of the day/week. All these instructional strategies will help develop the problem solving abilities in students and enhance their conceptual understanding thereby making students enjoy and use math appropriately.

VI. Representing

This simply implies presenting mathematical concepts and procedures in various forms or ways to facilitate deeper understanding. For effective conceptual understanding, students should be able to represent mathematical ideas in multiple ways (such as numeric, geometric, algebraic, graphical, pictorial, etc.); connect and compare representations; and select and apply the appropriate representations in solving problems. This can be done by using appropriate representation including: concrete/manipulative materials, pictures, diagrams, graphs, tables, words, numbers, and symbols. All these representations are particularly valuable tools which represent thinking and allow the exploration of abstract concepts in a hands-on, concrete way. Representation of mathematical ideas and relationships in various forms facilitate students' learning thereby enabling connections making and flexibility development in their thought about mathematics. A teacher can therefore develop students' conceptual understanding by utilising multiple representations during instruction to explore and represent math concepts as these will enable students think expansively and for themselves, e.g. $\frac{1}{2} + \frac{1}{2} = 1$

A teacher can achieve this by using various appropriate models when presenting or reviewing a concept, utilizing concrete materials when introducing new concepts; posing questions requiring the use of various representations by students at each level of conceptual development, e.g. from concrete to visual and to symbolic level as they continually revisit each stage in order to make connections. Conceptual ideas are not developed once in students but takes repeated and consistent effort in various contexts before it can be developed. Hence, students should be given sufficient time to internalize and solidify their concept understanding at each conceptual stage (TIPS4RM, 2005; Cystal, 2016).

VII. Communicating

Communicating one's thinking and reasoning is key in mathematics. Communication enables students to develop and share their understanding by using the appropriate language precisely and effectively (Tips4rm, 2005) in communicating mathematically. Thus, mathematical thinking can be communicated in various ways such as: orally, visually and in writing using appropriate mathematical vocabulary. It involves using multiple representations (such as: numbers, symbols, pictures, graphs, diagrams and words) to express mathematical concepts and solutions. Communication also enhances students' reflection and clarification of ideas, relationships and mathematical arguments. During math instruction, students are expected to use correct and appropriate mathematical language and vocabulary in explanations and in responses as appropriate for the audience, e.g. simplify, evaluate, factorise, expand, solve, etc. They should also be able to present thinking and arguments in a logical and organized manner; respond clearly with sufficient details for easy understanding of their thinking, interpret and summarise information from charts and graphs; and use symbolic language correctly, e.g. '=', $<$, $>$, $\leq$, $\geq$, $\angle$ for angles, etc.; and communicate mathematical learning by combining various representations, e.g., words with diagrams, charts with verbal descriptions, and so on. To develop and enhance this process, teachers should model the correct usage of mathematical symbols, conventions, vocabulary, notations and also encourage students to do likewise. This will in turn enhance the students' conceptual understanding in math class.

**Conclusion**

Meaningful math learning results from emphasizing deeper understanding of math concepts through detailed process and steps rather than focus on algorithm and procedures memorization and shortcuts in math class. Students should therefore be assisted in developing a deeper understanding of math concepts in order to improve both math instruction and students' learning.

This article therefore considered the development of secondary school students' conceptual understanding via mathematical processes. The seven mathematical processes considered include: reasoning, reflecting, selecting tools and computational strategies, connecting, problem solving, representing and communicating respectively.

**Recommendation**

It is recommended that teachers should emphasize conceptual understanding which is simply deep knowledge of concepts and processes rather than focus on algorithm memorization and shortcuts by emphasizing the mathematical processes considered. This will go a long way in facilitating students' deeper understanding of math, improving their performance and thereby producing students with appropriate math skills that can be applied in any field of learning, thus making them become an effective citizen of the larger society.

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