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Abstract
Problem solving skills are learned essential skills requiring instruction and adequate practice. Mathematics teachers at any educational level (primary, secondary and tertiary) can develop and enhance these skills by choosing skills that will keep students engaged in the problem-solving process, structuring and discussing the stages of problem-solving process, and effective teaching practices. Creating a foundation, questioning techniques, modelling of problem-solving skills, guiding students' problem-solving skills and constant practicing of problem-solving tasks are some effective teaching practices and strategies that teachers can engage in that will make good problem solvers at any educational level in mathematics. Developing problem solving skills in students thus makes them active in and out of class, improves their learning in mathematics, and makes them become confident and competent problem solvers in mathematics and in the real world.

Introduction

Life consists of problems of different severity or complexity. Some may be simple to solve while others may be difficult to solve. Developing a solution to such problems is essential. Today's mathematics requires more than computational skills. It requires the ability to reason and think mathematically and apply the same in solving familiar or unfamiliar problems that may arise in

the future (Van de Walle, 2004). In most mathematics curriculums, problem solving is incorporated into the contents for the purpose of developing the skills to problem solve. The significance of mathematical problem-solving learning results from the general belief that mathematics is basically about reasoning and not mere memorisation. Mathematical problem solving in curriculum usually comes in two forms namely: true problems or exercises but most students solve problems as mere exercises due to lack of certain skills. Toffler (2019) defined problem solving as the ability to identify the key questions in a problem, develop possible plans for solving, follow through on those plans, and evaluate both the success of the plan and the solution. These represent the processes of resolving a problem. Problem solving entails the development of understanding and detailed explanation of processes used in arriving at solution(s), and not mere remembering and application of a set of procedures previously acquainted with. Its significance involves developing sound understanding of mathematical concepts, becoming more engaged, and appreciating the relevance and usefulness of mathematics (Wu & Zhang, 2006).

Bommel and Palmer (2015) viewed problem solving's role in three ways: as a context, an art and a skill (p. 78). The role of problem solving as a context implies utilizing problem solving to reach a goal and facilitate learning of other valuable skills. It is utilized as a method of justifying the teaching of mathematics, for arousing and motivating students' interest in mathematics and as a means of reviewing prior knowledge and new concepts and skills. Problem solving as an art implies teaching and learning about the processes of solving problems and not just learning it as a skill. Problem solving as a skill is viewed as an essential element whose development will enhance the solution of problems within and outside mathematics. Burns (2000) maintained that to enhance mathematical problem solving, teachers should focus more on the students' thinking processes rather than the direct instruction. Their focus should be on facilitating and guiding the students through problem solving processes, provision of more time for students' understanding of the problems, self-search for strategies and solutions, evaluation and reflection on results obtained.

According to Woodham (2014), to become confident and competent in problem solving demands skills and experience of student's which teachers can help to develop. Problem solving skills can thus be developed when a solver can successfully implement the processes of solving a problem (Ozrecberoglu & Caganaga, 2017). This made Wood (2017) regard problem solving as a lifelong essential skill needed to be developed. Problem solving skills are beneficial to everyone who encounters problems daily (Dendane, 2009). It enables one to identify and efficiently solve

problems in a timely manner without encountering much difficulty. As problem solving is a major goal in mathematics curriculum, many mathematics educators recognise the importance of developing problem solving skills in students. Mills and Kim (2017) emphasized that problem solving skills are not developed naturally but taught to students in a way that can be transferred across multiple settings and contexts. Dendane (2009) believed that when genuine mathematical problem solving is well facilitated, it may help students: gain a deep and better understanding of mathematical concepts; develop critical thinking and reasoning skills; develop and improve the generic ability to solve real life problems. By implication, students are trained for life beyond school with mathematics problem solving skills (Klerlein & Harvey, 2019). In this article, how to develop problem solving skills in students is considered. Hence, the definitions of skill, mathematical skills, and problem-solving skills are unpacked next.

Skill Definition

In Longman Dictionary of Contemporary English (2014), skill is an ability to do something well especially because you have learned and practised it. That is, it is the learned power of doing something competently (Merriam Webster dictionary, n.d.). Tomaszewski (2020) described skills as the expertise developed to perform a task or a job.

Mathematical Skills

Mathematics skills manifest in children even before the beginning of formal school which they develop in the day-to-day activities and interactions such as addition and subtraction.

Other basic math skills learnt by students in schools include the following:

a) Number sense.
b) Representation of mathematical ideas (in words, pictures, symbols etc.)
c) Spatial sense (geometry, ideas of shapes, size, space, direction, and movement).
d) Measurement (finding the length, weight, and height of objects)
e) Estimation (less than, more than, etc)
f) Patterns (making predictions and logical connections).
g) Problem solving (Angelsophia, 2018).

Problem solving

Problem solving is one of the top basic skills which students learn as it helps them in developing analytical thinking skills aside necessary computational skills, estimation and approximation skills which enable students to apply math to everyday situations (Angelsophia, 2018). Problem solving

in mathematics can be challenging to teach and learn because it relies on many skills (Dandane, 2009) to achieve a satisfactory solution. Teachers encounter difficulties in teaching problem solving if their understanding of problem-solving process is limited. Likewise, students too encounter difficulties in learning genuine problem solving when their learning is limited to routine problems (exercises already solved in the past) which require little or no basic skills. Problem solving skill is a hard skill that is learnt through formal education of additional training (Tomaszewski, 2020). Hence, what is unpacked next is what problem-solving skills are?

What is problem solving skills?

Fletcher (2019) described problem solving skills as "mental processes that allow one to take on a problem, choose the best of many problem-solving techniques for that particular situation, and think through the steps to find a solution". That is, problem solving skill is what enables a solver to identify and understand a problem, search for the different various strategies to solve the problem with the least difficulty possible in the most effective way. Problem solving skill also referred to the "ability or strategic competence shown students in understanding, selecting approaches and coping strategies as well as a complete model to find the solution of a problem" (Noprianilubis, Paijatan, Surya & Syahputra, 2017, p.132). It is viewed as an essential life skill entailing variety of mathematical processes such as: analyzing, interpreting, reasoning, predicting, evaluating and reflecting. Hence, Noprianilubis, Paijatan, Surya and Syahputra (2017) concluded by describing problem solving skill in mathematics as the ability of students to solve problems by observing the process of finding answers based on the step by step problem solving namely: understand the problem; devise a plan; solve a problem in accordance to plan; look back/ reexamine the results obtained. By implication, problem solving skill is the ability to utilize the general problem solving processes in solving a particular problem and achieve a desired result or obtain an optimal solution to the problem. That is, the skill required in implementing each stage of the problem solving processes to resolve the problem. According to Chaudhry and Rasool (2012), problem solving skills are significant requirements to attain individual's desired goals in life which could not be underestimated. This could be a reason why problem solving was emphasized in the mathematics curriculum as an essential element to be developed in students.

Types of problem-solving skills

The effective learning of mathematical problem solving requires lots of skills (Dendane, 2009). Students encounter difficulties in mathematical problem solving at each stage of the process. Some

students have difficulty in reading the problem statement with understanding, while some encounter challenges in extracting information from the problem statement. Thus, to successfully problem solve in mathematics, there is need to acquire some skills that will enhance becoming good problem solvers. This is because, without a good understanding of the problem and its processes, there may be difficulty in making any progress in mathematical problem solving. A basic step to successful problem solving is the development of problem-solving skills and these skills have to be taught to students explicitly. Dendane (2009) maintained that a major goal of teaching mathematical problem solving is to enable students develop a generic ability in solving real life problems and apply maths in real life situations. Two kinds of mental skills required for problem solving are given as follows:

1. Critical thinking skill
2. Logical reasoning skill.

1. Critical Thinking Skill

Joe and Jonathan (2020) described critical thinking as the ability to think clearly and rationally about what to do. It is the reflexive and independent thinking for logical connections understanding between problem information and the systematic solution to problems. That is, the ability to think and solve problems effectively using knowledge, facts, and data. Alcantara and Bacsa (2017) regarded critical thinking as "the mental processes, strategies and representations people use to solve problems, make decisions and learn new concepts" (p.21). They maintained that critical thinking skills are skills that allow an individual to solve problems through analysis and synthesis and evaluation of information to increase students' success. Every problem-solving process step requires critical thinking skill ranging from understanding and extracting information from problem statements, resolving problems and analyzing solutions obtained (Dendane, 2009). Snyder & Snyder (2008) recognized the importance of teaching content to students, yet they rated higher the importance of the material learning process. They stressed that it is essential that teachers teach students how to think rather than 'teaching them what to think'. Also, students think only during the teaching process. Critical thinking is not an inborn ability, rather it is a learned skill that needs development, constant practicing, and continual integration in the curriculum for students' active learning. That is, a skill that can be learnt via teaching and constant practice (Snyder & Snyder, 2008) that will enable students to solve problems effectively. According to

Snyder and Snyder (2008), critical thinking can be aroused through utilizing questioning techniques which entails the utilization of higher order thinking skills such as analysis, synthesis, and evaluation in solving problems and thinking instead of just memorizing the fact. Most of the classroom instructions in mathematics emphasise content learning which is mainly teacher centered thereby debarring students from exploring the content, analyzing resources that could be useful in solving problems and applying content learnt. Thus, critical thinking is essential as it helps students determine how best to tackle and solve a problem, answer a question, and handle a situation (Burroughs, n.d.).

2. Logical Reasoning Skills:

Reasoning is important to knowing and doing of mathematics. Reasoning in mathematics is defined as the process of applying logical and critical thinking to a mathematical problem for working out the correct strategy to utilise (and as importantly, not to use) in reaching a solution (MrBeeTeach, 2020). Reasoning skill entails the utilisation of good sense to think through a problem and justify the thoughts, actions or opinion utilised in the process of getting from the problem point to the solution point (Burroughs, n.d.).In essence, logical reasoning skill involves the ability to think through a problem logically, to identify relevant and irrelevant information in order to arrive at a solution and to also justify the solution got (National Curriculum, n.d).Simply put, reasoning is the ability to make sense of mathematical ideas and apply them in solving problems in familiar or unfamiliar situations. Reasoning enhances student's confidence, makes them independent mathematical thinkers and grants them the ability to confront problems at any time in or outside mathematics. Thus, Dendane (2009) believed that logical reasoning is important in achieving success in maths in general and particularly problem solving. Two types of logical reasoning in problem solving include:

I. Inductive reasoning
II. Deductive reasoning

I. Inductive reasoning:

This involves pattern identification and establishment of relationships between mathematical objects useful in making educated or intelligent guesses, testing them and making generalizations (Dendane, 2009; Resnick, 1987).

II. Deductive Reasoning:

This involves making a logical arrangement, using mathematical definitions, rules, and theorems to draw conclusions and applying generalizations to specific situations (Dendane, 2009; Resnick, 1987).

Problem solving skills in this context simply implies the skills needed or required by the students to effectively solve a problem and work on a mathematical task via the problem-solving process to obtain a satisfactory solution.

This article adapted Woodham's (2014) useful problem-solving skills to be developed in students in mathematics. They include:

a) Trial and improvement
b) Working systematically
c) Pattern spotting.
d) Working backwards
e) Visualising
f) Conjecturing

The skills are useful and utilized at stage 2 and 3 of Polya's problem solving process (which is devising a plan and solving the problem), that is, the skills involved in identifying the strategy (ies) for problem solving task and solving such.

The skills are expounded as follows:

1. Trial and improvement: This is simply a trial-and-error process which involves attempting different ways in tackling a particular problem in order to gain insight into the problem context and to discover how to better tackle the problem next (Woodham, 2014). Thus, in the words of Woodham (2014), trial and improvement 'involves trying something out, which will always give more insight into the context and therefore gives the solver a better idea of what to try next'.
2. Working systematically: This implies working methodically and efficiently on a problem such that it will display the pattern or system being used for better clarification and understanding.
3. Pattern spotting: The identification of pattern and occurrence reasons for insight into mathematical structure and for deeper conceptual understanding of concepts.

4. Working backwards: This implies tackling a problem from the rear. According to Woodham (2014), starting from the end of a problem can be an efficient way of solving a problem.

5. Visualizing: This involves "picturing what is happening in one's mind's eye" (Woodham, 2014). It requires that students picture familiar situations or settings and describe what they see. visualizing entails representing mathematical problems with pictures and other schematic or visual representations such as diagrams, tables, graphs, etc. to show relationships among the parts of a problem (Warger,2018).

6. Conjecturing: This implies guessing based on known information. In other words, Woodham (2014) referred to this as asking, "What if…?" questions depending on the understanding of the problem's structure.

Ways and Strategies of Developing Problem-Solving Skills

Teachers require many skills and experience to teach and help students develop problem solving skills and become confident and competent problem solvers (Pennant, 2014). The complex nature of problem solving requires teachers' better understanding of it to be able to develop effective classroom activities and tasks that will aid in teaching it and developing the problem-solving skills in students. For students to develop problem solving skills, teachers need to discuss these skills in context and give room for practicing of the skills in classroom. This will grant students the opportunity to become confident and competent problem solvers (Woodham, 2014).

Some of the ways of doing this includes the following according to Pennant (2014):

1. The Choice of Task:

Teachers need to carefully choose tasks that will engage learners in the problem-solving process. Pennant (2014) suggested students' exposure to tasks that will enhance exploration from their level of understanding. Teachers can choose contents the students are familiar and more confident with but should ensure that the problem tasks are such that will build on specific skill of problem solving.

2. Structuring and discussing the stages of the problem-solving process:

Teachers need to teach and emphasize in detail the four basic stages of problem-solving process as propounded by Polya (1945) cited in Akinsola and Awofala (2008, 2009) and Awofala (2011, 2014, 2017) which are:

Stage 1- Understanding the problem.

Stage 2- Devising a plan.

Stage 3- Solving the problem.

Stage 4- Looking back.

There is need for detailed emphasis and explanation of each stage in mathematics context to develop students' skills for competence in problem solving (Pennant, 2014; Awofala, Balogun & Olagunju, 2011; Awofala, Fatade & Olaoluwa, 2013).

3. Effective Teaching Practices

The development of problem-solving skills requires explicit and repeated provision of opportunities to learners. There is the need for teachers to recognize the fact that problem solving processes develop over time and are significantly improved via effective teaching practices.

These teaching practices and strategies that will make good problem solvers at the pre-tertiary (primary and secondary) levels involve the following:

1. Creating a foundation: Teachers should endeavor to always begin the class by reviewing a previous knowledge related to the problem-solving topic which can emanate from previous lessons or assignments or real-life situation, etc. (12 solid strategies, 2019). This will enable them recall ideas, information, and facts relevant to the topic. This will also enable the teachers to assess students' background knowledge and then take a decision on the best approach to the problem considered (SkillsYouNeed, 2020).
2. Questioning Techniques: The use of questioning techniques enhances students' active participation in the learning process (Snyder & Snyder, 2008). To develop problem solving skills, teachers should start by asking a right question, not a "yes or no" question (12 solid strategies, 2019) but those that will lead students to think, brainstorm and understand what the problem is asking or all about and how to go about solving the problem.

Sample questions could include the following:

- What is the question asking for?
- What are the key words in the problem?
- What do you think you can do first to solve the problem?

Such questions provoke students' thought process and make them to "think about their thinking" so as to interpret and understand the given problem. Snyder and Snyder (2008) maintained that questions involving thinking skills require time and patience. Hence, after asking questions, teachers should exercise patience for like 8-12 seconds to enable the students think critically about

the problem before demanding for their responses on the problem or question asked. This will create the climate of freedom and openness that will encourage increased contribution and input of students' ideas to the lesson.

3. Modelling problem solving skills: The development of problem-solving skills in students require that teachers need to display how they solve problems, that is, they need to demonstrate the skill adequately and appropriately in their lessons for students to emulate the behaviour. Snyder and Snyder (2008, p. 94) maintained that students needed to be taught how to think and follow through the general processes of solving problems before their application since "students are not born with the ability to think critically, and their prior learning experiences often do not require them to think critically".
4. Guiding students' problem solving skills: Comfortable learning environment should be created by teachers to enable learners become active participants rather than just memorise or recall information which characterized passive learning. Snyder and Snyder (2008) suggested that teachers could do this by utilizing peer groups as students learn better in teams and feel comfortable with learning, because most times it gives them room and freedom to obtain information, express themselves and ask questions bothering their mind about any problem solving topic. They maintained further that this problem solving technique of collaboration learning enhances critical thinking and development of problem solving skills.

   Facoine (2007) as cited in Snyder and Snyder (2008) outlined six effective thinking and problem solving steps tagged 'IDEALS'. He stated that guiding students' problem solving skills could be done by assigning the students to two-person teams comprising the problem solver and the peer coach respectively. The peer coach utilizes the "IDEALS" to ask and guide the problem-solver through a problem solving activity.
   The "IDEALS" entails: Identify, Define, Enumerate, Analyse, List and Self-Correct.

   I – Identify the problem: What is really the problem or what is the question asking?

   D – Define the context: What are the key words or facts given in the problem?

   E – Enumerate the choices: What are the possible alternative strategies?

   A – Analyse options: What is the best course of action or appropriate strategy?

   L – List reasons explicitly and solve: Why is this the best course of action or appropriate strategy to the solution of the problem?

S – Self-correct: Look at it again, what is missing?

5. Constant Practicing of problem solving: Teachers should always expose students to more problem solving activities. That is, students should constantly be provided with problem solving tasks to develop their problem solving skills. The more they solve diverse problems involving variety of strategies, the more developed their problem solving skills and improved. Teachers should also emphasise the significance of the solution process and not the specific answer because solutions to problems may take a variety of forms. Thus, students should be given more room and opportunity to explore the mathematics adequately and freely.

Developing Mathematical Problem solving skills in Tertiary Education students

Every individual at any educational level (primary, secondary and/or tertiary) require problem solving skills to solve any challenging problem in any situation in school or out of school. Hence, the reason why the problem solving skills needed to be developed in students right from primary school through to their tertiary level. Unfortunately, these skills are not taught via the traditional curricular being used in most universities (Chaudhry & Rasool, 2012) and these skills cannot be underestimated and compromised. Chaudhry and Rasool, (2012) stressed the importance of problem solving skills and their development in tertiary institution students prior to their graduation and entry into the real world where it will be demanded of them to demonstrate practically to solve real life problems.

Woods et al. (1997) carried out a study titled "Developing problem solving skills: The McMaster problem solving program", where they utilized some methods which they found effective for developing students' problem solving skills in their 25-year project in Chemical engineering. These methods can also be useful and applicable to developing problem solving skills in mathematics students at the tertiary level. They include the following:

a) The use of open-ended problems. This entails asking/posing open-ended problems that will provoke the students' critical thinking that will enable them solve the problem.
b) Teacher's display/demonstration of approach to problem solving, i.e. showing students how to solve problems practically via worked problems and giving out many sample solutions.
c) Active engagement of students to discover the problem solving process such as asking students to solve problems on the board.

d) Utilizing peer collaboration to demonstrate the problem solving process.
e) Giving explicit suggestions about how to solve problems such as selecting the appropriate strategies from variety of alternatives to be applied in solving problems like; drawing diagrams, making a list, solving simpler equations, translating the problem to mathematical symbols and equations before inputting numerical values, etc.

It is important to point out to students that aside communication skills; capacity to learn new skills and procedures; and team work, employers of labours look out for problem solving skills as important quality when hiring tertiary institutions' graduates. It is therefore important to develop these skills in undergraduates for application in solving real life practical problems or whenever they encounter any issue that are new to them. More so, developing these skills makes students active learners in class and improves their learning in mathematics.

Conclusion

Problem solving skills are essential in building students' confidence, making them good decision makers and equipping them for exploring difficult issues and developing suitable responses to these issues. In mathematics, problem solving is recognized as a major goal in the curriculum, it is thus important to develop the skills in students at all levels of education; primary, secondary and tertiary. These skills are not naturally developed but can be taught to students in such a way that will enable them have a clearer insight of mathematical concepts, develop critical thinking and reasoning skills and also improve their generic ability to solve mathematical and real life problems. Critical thinking and logical reasoning are essential problem solving skills necessary in understanding a problem and comprehending the appropriate strategy (ies) useful in solving particular problems. Some other skills needed to be developed in students to effectively solve problems and work on mathematical tasks via problem solving process to obtain satisfactory solution(s) include trial and improvement, working systematically, pattern spotting, working backwards, visualizing and conjecturing amongst others.

Developing these skills in students at any educational level requires teacher's better understanding in developing effective classroom activities and tasks, discussing and modelling problem solving process and skills, and adequate practicing of the skills in the classroom. These will develop and enhance students' problem solving skills in mathematics, thus making them become confident and competent problem solvers in and outside mathematics.

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